MEACS2015 IOP Publishing
IOP Conf. Series: Materials Science and Engineering **124** (2016) 012081 doi:10.1088/1757-899X/124/1/012081

Calculation of the surface effect in the ferromagnetic conductor with the harmonic electromagnetic field

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Abstract. The authors of the paper have obtained formulas for analytical calculation of the constants with the harmonic electromagnetic field, which characterize the surface layer (a skin layer) of the ferromagnetic conductors in case of heating and nonlinear magnetic properties, which can be used for practical calculation of the electromagnetic screens, rotors of the electrical machines and inductive heating installations. A nonlinear dependence of the magnetic induction on the magnetic tension of the ferromagnetic conductor is replaced by one or two linear sections. It is considered that the skin layer of the conductor has constant quantities of the specific conductivity and averaged temperature. Linear electrodynamics equations are solved for the conductive half-space. Parameters of the ferromagnetic conductor's surface layer are calculated: magnetic permeability, the thickness of the skin layer and its averaged temperature, exposure time of the electromagnetic field on the conductor with the established maximum temperature on the conductor's surface, pressure of the field on the conductor and its resistance, inductivity of the internal magnetic field in the conductor, the thermal energy capacity. The methods credibility is confirmed with the concurrence of the resistance and inductivity of the ferromagnetic conductor with analogous quantities from other methods.

1. Introduction

It is often necessary to calculate parameters along with the calculation of the electro energetic equipment, which characterize a surface effect in ferromagnetic conductors with the harmonic electromagnetic field (EMF). The calculation are simplified with the parameters, which characterize the conductor's surface layer (a skin layer). These parameters include the thickness of the skin layer and its temperature, EMF pressure on the conductor, and also the conductor resistance and the internal magnetic field inductivity [1-5, 7, 8]. A numerical computation of these parameters, taking into account the heating of the skin layer and the nonlinear magnetic properties of the ferromagnetic conductors, is concerned with huge machine time expenses, and calculations with the use of the known formulas are quite approximate and not enough well-grounded [1-5].

Thus the analytical calculation with harmonic EMF parameters of the skin layer and ferromagnetic conductors in case of its heating and nonlinear magnetic properties is one of the most important problem at present.

The main idea of this paper is the methods development, taking into account heating of the skin layer and its nonlinear magnetic properties, for the practical analytical calculation of the skin layer properties of ferromagnetic conductors in case of harmonic EMF.

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To develop the methods the following assumptions are included:

1. The conductor skin layer is characterized by such constant quantities as specific conductivity γ (1/Ω·m), averaged temperature θ (°C), specific thermal capacity C (J/kg·°C), thermal conductivity λ (W/m·°C) and volume density ρ (kg/m³).
2. The skin layer heating is quite short-term for its processing without heat exchange with the environment [4]. In this case maximum temperature of the ferromagnetic conductor θ_m does not exceed Curie temperature $\theta_K \approx 750$ (°C), thus, the absolute (static) and differential magnetic conductivities do not depend on the temperature [2].
3. The hysteresis losses are insignificant and are not taken into account in comparison with the eddy currents losses [2]. Thus, one-digit magnetization curve $B(H)$ of the ferromagnetic material of the conductor, which is symmetrical to the coordinate origin, is described as a straight line in the first approximation (fig. 1, dependence 1), and in the second approximation – as a two linear section (Figure 1, dependence 2).
4. Conductors sizes and surface curvature radiuses exceed greatly the thickness of the skin layer, thus we will consider the surface effect as penetration of the one-dimensional plane electromagnetic wave into the conductive ferromagnetic half-space [2, 4].
5. Harmonic EMF is specified by magnetic tension on the surface of the conductor, where $z=0$ (the surface of the half-space):
$$H_S(t)=H_m\sin(\omega t), \tag{1}$$
t is time; $\omega=2\pi f$ is angular frequency (1/s).
6. The duration of EMF, to which conductor τ is exposed, greatly exceeds the period of the tension alteration (1), therefore $\tau \gg T = 2\pi/\omega$. Thus the usage of the envelope dependences for the active (root-mean-square) current density, maximum tension and maximum induction is fully well-founded.

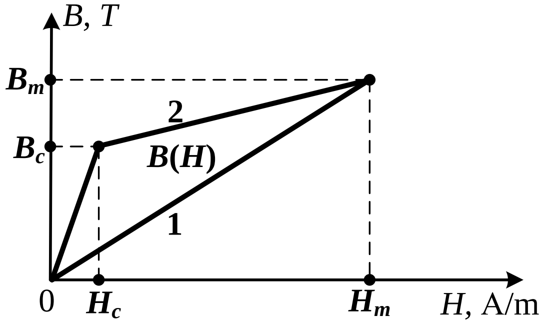

Figure 1. Approximate dependences $B(H)$ of the ferromagnetic conductor: B_m, H_m and B_c, H_c – are maximum and intermediate values of the magnetic induction, magnetic tension.

2. Methods

For the one-dimensional plane electromagnetic wave in the conductive half-space the design equation for magnetic tension $H_{y1,2}(z,t)$ and induction $B_{y1,2}(z,t)$ can be described as [2, 4]
$$\frac{\partial^2 H_{y1,2}(z,t)}{\partial z^2}=\gamma\frac{\partial B_{y1,2}(z,t)}{\partial t}=\mu_{d1,2}(z,t)\gamma\frac{\partial H_{y1,2}(z,t)}{\partial t} \quad . \tag{2}$$
Then the current density will be
$$\delta_{x1,2}(z,t)=-\frac{\partial H_{y1,2}(z,t)}{\partial z} \quad , \tag{3}$$

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where the differential magnetic permeability for dependence 1 (Figure 1) is constant and equals static magnetic permeability:

$$\mu_{d1}(z,t) = dB_{y1}(z,t)/dH_{y1}(z,t) = \mu_1(z,t) = B_{y1}(z,t)/H_{y1}(z,t) = \mu_m = B_m/H_m\,. \quad (4)$$

And for dependence 2 (Figure 1) the differential magnetic permeability equals

$$\mu_{d2}(z,t) = \frac{dB_{y2}(z,t)}{dH_{y2}(z,t)} = \begin{cases} \mu_c = B_c/H_c & if \;\; 0 < |H_{y2}(z,t)| < H_c; \\ \mu_d = (B_m - B_c)/(H_m - H_c) & if \;\; H_c < |H_{y2}(z,t)| < H_m \end{cases} \quad (5)$$

with the static magnetic permeability in accordance with (5)

$$\mu_2(z,t) = \frac{B_{y2}(z,t)}{H_{y2}(z,t)} = \begin{cases} \mu_c & if \;\; 0 < |H_{y2}(z,t)| < H_c; \\ \mu_d + (\mu_c - \mu_d) H_c / |H_{y2}(z,t)| & if \;\; H_c < |H_{y2}(z,t)| < H_m. \end{cases} \quad (6)$$

With equations (1-4) according to [2, 4] for dependence 1 (fig. 1) let us calculate tension

$$H_{y1}(z,t) = H_m \exp(-z/\Delta_1) \cdot \sin(\omega t - z/\Delta_1) \quad (7)$$

and current density

$$\delta_{x1}(z,t) = \sqrt{2}(H_m/\Delta_1)\exp(-z/\Delta_1) \cdot \sin(\omega t - z/\Delta_1 + \pi/4) \quad (8)$$

in case of equivalent depth of the penetration of harmonic EMF into the conductive half-space with magnetic permeability (4)

$$\Delta_1 = \sqrt{2/\mu_m \gamma \omega}\,. \quad (9)$$

As a result, on the basis of (7, 8) for straight line 1 (fig. 1) envelope dependences on depth z for the maximum tension equal

$$H_1(z) = H_m \exp(-z/\Delta_1)\,. \quad (10)$$

And for the active (root-mean-square) current density they are equal to

$$\delta_1(z) = (H_m/\Delta_1)\exp(-z/\Delta_1)\,. \quad (11)$$

For dependence 2 (Figure 1) in accordance with (5, 6, 9-11) envelope dependences in the function from z are described as

$$H_2(z) \approx H_m \exp(-z/\Delta_1) \text{ for the maximum tension,} \quad (12)$$

and for the maximum induction they are

$$B_2(z) = \begin{cases} B_c + \mu_d [H_2(z) - H_c] & if \;\; 0 < z < c; \\ \mu_c H_2(z) & if \;\; c < z < \infty; \end{cases} \quad (13)$$

for the active current density they equal

$$\delta_2(z) \approx (H_m/\Delta_2)\exp(-z/\Delta_2), \quad (14)$$

where the thickness of the skin layer can be found according to the following formula:

$$\Delta_2 = \sqrt{2/\mu_p \gamma \omega}\,. \quad (15)$$

In case of design magnetic permeability it is

$$\mu_p \approx \begin{cases} \mu_c & if \;\; 0.5H_m < H_c; \\ \mu_d + (\mu_c - \mu_d) H_c / 0.5H_m & if \;\; H_c < 0.5H_m \end{cases} \quad (16)$$

and the distance from the conductor surface is

$$c = \Delta_1 \cdot \ln(H_m/H_c). \quad (17)$$

At that $B_2(0) = B_m; B_2(c) = B_c;\; H_2(0) = H_m; H_2(c) = H_c$.

Using (5, 9, 12-17) for dependence 2 (Figure 1) the formulas for the calculation of the maximum energy of the magnetic field inside the conductor [2, 4] are

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$$W_m = l_x l_y \int_0^\infty \left[\int H_2(z)\, dB_2(z) \right] dz = \left[1 + \left(H_c^2 / H_m^2 \right) \left(\mu_c / \mu_d - 1 \right) \right] H_m^2 l_x l_y \sqrt{\mu_d^2 / 8\mu_m \gamma\omega}, \tag{18}$$

and the capacity of the thermal energy is

$$P = l_x l_y \int_0^\infty \left[\delta_2(z)^2 / \gamma \right] dz = H_m^2 l_x l_y \sqrt{\mu_p \omega / 8\gamma}\ , \tag{19}$$

where *lx*, *ly* – are sizes of the conductor on axis x and y.

In the conductor along axis x there is induced or created by the external source harmonic current, which, in accordance with (1), can be found using Ampere's circuital law [4]:

$$i(t) = l_y H_S(t) = l_y H_m \sin(\omega t) = I_m \sin(\omega t). \tag{20}$$

The function of the linkage of the magnetic flux in the conductor with current (20) can be found using equation (11),

$$K(z) \approx \left(l_y / I_m \right) \int_z^\infty \delta_1(z)\, dz = \exp\left(-z / \Delta_1 \right), \tag{21}$$

Then, in accordance with (4, 5, 9, 13, 21), the maximum interlinkage for the magnetic field inside the conductor equals [4]

$$\Psi_m = l_x \int_0^\infty B_2(z) K(z)\, dz = 0{,}5 \mu_d H_m \Delta_1 l_x \left[1 + \left(H_c / H_m \right) \left(2 - H_c / H_m \right) \left(\mu_c / \mu_d - 1 \right) \right]. \tag{22}$$

For current (20) from the equations described for energy (18), interlinkage (22) and capacity (19) it follows that

$$W_m = 0.5 I_m^2 L_w; \quad \Psi_m = I_m L_\psi; \quad P = 0.5 I_m^2 R.$$

Let us calculate inductions

$$L_\psi = \sqrt{\frac{\mu_d^2}{2\mu_m \gamma\omega}} \left[1 + \frac{H_c}{H_m} \left(2 - \frac{H_c}{H_m} \right) \left(\frac{\mu_c}{\mu_d} - 1 \right) \right] \cdot \frac{l_x}{l_y}; \ L_w = \sqrt{\frac{\mu_d^2}{2\mu_m \gamma\omega}} \left[1 + \frac{H_c^2}{H_m^2} \left(\frac{\mu_c}{\mu_d} - 1 \right) \right] \cdot \frac{l_x}{l_y} \tag{23}$$

and resistance of the conductor

$$R = \sqrt{\mu_p \omega / 2\gamma} \cdot l_x / l_y\ . \tag{24}$$

At that $L_\psi \geq L_w$ and for linear magnetization curve $B(H)$, which has $\mu_m = \mu_d = \mu_c$, $L_\psi = L_w$ as well.

From the resulting inductance of the internal magnetic field of the conductor let us find the average value of L_ψ and L_w:

$$L = \left(L_\psi + L_w \right) / 2 = \sqrt{\mu_m / 2\gamma\omega} \cdot l_x / l_y\,. \tag{25}$$

Then, in accordance with (24, 25) we can deduce the equation for the conductive half-space with the linear magnetization curve in case of $\mu_m = \mu_p$: $R = \omega L$ [4].

On the basis of equations (11, 13) let us calculate the EMF maximum pressure exerted on the conductor, which is directed along axis *z* [2]:

$$\sigma_m \approx \int_0^\infty \left[\delta_1(z) B_2(z) \right] dz = 0.5 B_m H_m \left(1 + B_c / B_m - H_c / H_m \right). \tag{26}$$

Thus, nonlinearity of dependence $B(H)$ increases pressure σ_m: for rectangular $B(H)$, when $H_c = 0; B_c = B_m$, we obtain $\sigma_m = B_m H_m$, and for linear $B(H)$ with $H_c / H_m = B_c / B_m$ we have correct value $\sigma_m = 0.5 B_m H_m$ according to [2].

Without heat removal into the environment the equation of conductor's temperature $\theta_T(z,t)$ and current density (14) will be [5]

$$\rho C \frac{d\theta_T(z,t)}{dt} = \frac{\delta_2(z)^2}{\gamma} + \lambda \frac{\partial^2 \theta_T(z,t)}{\partial z^2}. \quad (27)$$

With temperature $\theta_T(z,0)=\theta_0; \theta_T(\infty,t)=\theta_0$ and (14, 15) the solution of equation (27) is

$$\theta_T(z,t) = \theta_0 + \left(H_m^2/4\gamma\lambda\right)\left[\exp\left(4\lambda t/\Delta_2^2\rho C\right)-1\right]\exp\left(-2z/\Delta_2\right). \quad (28)$$

Therefore maximum temperature θ_m of the conductor with nondimensional parameter χ and duration of the EMF exposure τ will be equal to

$$\theta_m = \theta_T(0,\tau) = \theta_0 + \frac{H_m^2}{4\gamma\lambda}\left[\exp(\chi)-1\right]; \; \chi = \frac{4\lambda\tau}{\Delta_2^2\rho C}; \; \tau = \frac{\Delta_2^2\rho C}{4\lambda}\cdot \ln\left[1+\frac{4\gamma\lambda(\theta_m-\theta_0)}{H_m^2}\right], \quad (29)$$

where θ_0 is initial temperature of the conductor.

Let us consider that the thickness of the skin layer equals Δ_2 (15), thus with (28, 29) the equation for the calculation of averaged temperature θ with specified conductivity γ [2, 5] is

$$\theta = \frac{1}{\tau}\int_0^\tau \left[\frac{1}{\Delta_2}\int_0^{\Delta_2}\theta_T(z,t)dz\right]dt = \theta_0 + \frac{0.108H_m^2}{\gamma\lambda}\left[\frac{\exp(\chi)-1}{\chi}-1\right]; \; \gamma = \frac{\gamma_0}{1+\alpha[\theta-\theta_0]+\beta[\theta-\theta_0]^2}, \quad (30)$$

where γ_0 is the quantity of the specified conductivity with temperature θ_0; α, β are the temperature constant coefficients.

3. Results

The initial data for the ferromagnetic conductor from [2] are shown in table 1. With correlations (1-30) the skin layer parameters of the ferromagnetic conductor were found (Table 2).

Table 1. A ferromagnetic cylinder conductor made of structural steel with diameter D = 0.1 (m) and length 1 (m)

l_x	$l_y = \pi D$	θ_0	ρ	C	λ	μ_0
m	m	°C	kg/m^3	J/kg·°C	W/m·°C	H/m
1	0.314	20	7800	575	42.5	$4\pi\cdot 10^{-7}$

γ_0	α	β	ω	θ_m	H_C	B_C	μ_c/μ_0
1/Ω·m	1/°C	1/°C^2	1/s	°C	A/m	T	–
$5\cdot 10^6$	0.0055	$9\cdot 10^{-6}$	314	700	4000	1.5	298.4

For testing of the developed methods the approximate formulas for the ferromagnetic conductors made of structural steel [2] were used:

$$\mu_S = 8130\mu_0\left(H_m/100\right)^{-0.894}; R = 1.32\sqrt{\mu_S\omega/2\gamma}\cdot l_x/l_y; L = 0.98\sqrt{\mu_S/2\gamma\omega}\cdot l_x/l_y, \quad (31)$$

where μ_S is the static magnetic permeability on the surface of the conductor with $z = 0$ and $4000 < H_m < 400000$ (A/m).

The results of the R and L calculation for (31) with θ and γ according to (30) are shown in Table 2. Resistance R and inductance L were also determined with the help of the computer modulation program 'Elcut' [6], when EMF creates the harmonic current, which runs in the cylinder conductor with diameter $D = 0{,}1$ (m) and length $l_x = 1$ (m). Besides, values $\omega, \gamma, \mu_p/\mu_0, \mu_m/\mu_0$ were used from Tables 1 and 2. At that μ_p/μ_0 were used for calculation of R, whereas μ_m/μ_0 were used for calculation of L as the average value of inductances $L_w, L_\psi, L_z = \mathrm{Im}(\underline{Z})/\omega$, which can be found with the help of the Elcut program, where $\underline{Z} = R + j\omega L_z$ is the conductor complex impendence ($j = \sqrt{-1}$). Parameters R and L found with Elcut [6] are shown in table 2.

Resistances R and inductances L calculated with equations (24, 25), [2] and [6] are approximately equal.

Table 2. Design parameters of the ferromagnetic conductor skin layer

H_m		kA/m	6	8	15.9	39.9	79.7	159.4	239.1	318.8
B_m		T	1.550	1.635	1.785	1.985	2.099	2.228	2.338	2.441
μ_m/μ_0		–	205.6	162.6	89.3	39.6	21.0	11.1	7.8	6.1
μ_p/μ_0		–	298.4	298.4	159.6	68.4	35.6	18.5	12.7	9.8
σ_m		kPA	6.05	9.27	22.55	65.56	139.2	292.7	454.2	623.3
τ		s	1.005	0.953	1.548	2.870	4.401	6.143	6.812	6.868
θ		°C	50.84	52.84	58.85	71.10	85.59	106.7	121.7	132.5
γ/γ_0		–	0.849	0.840	0.815	0.767	0.715	0.648	0.605	0.577
Δ_2		mm	2.000	2.010	2.791	4.395	6.309	9.192	11.47	13.38
P		kW	0.666	1.190	3.492	14.84	44.26	134.1	258.7	413.5
R		μΩ	375.0	376.9	279.9	189.0	141.2	106.9	91.7	82.4
L		μH	0.991	0.886	0.667	0.458	0.345	0.264	0.228	0.207
[2]	R	μΩ	414.4	366.3	273.6	187.0	142.1	109.5	94.5	85.1
	L	μH	0.979	0.866	0.647	0.442	0.336	0.259	0.223	0.201
[6]	R	μΩ	434.7	437.0	318.9	208.0	154.6	118.9	104.0	95.2
	L	μH	0.845	0.793	0.546	0.468	0.358	0.272	0.233	0.208

4. Conclusion

The analysis of the forms and calculation results allow formulating the following conclusions:

1. For the harmonic electromagnetic field the methods of analytical calculation of ferromagnetic conductor skin layer parameters in case of the considerable surface effect with the heating and nonlinear magnetic properties were developed. The methods can be applied for the induction heating devices, solid rotors of the electric machines and ferromagnetic screens design.
2. The formulas can be used for calculation of parameters of the nonferromagnetic conductors' skin layer with design magnetic permeability: $\mu_m = \mu_p = \mu_d = \mu_c = \mu_0$.
3. The credibility of the methods can be confirmed with the approximate equation of resistance *R* and inductance *L* of the ferromagnetic conductor with analogical quantities found by means of other methods.

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